Vivian Lam
2015 Summer Institute on China

Coordinate Plane: Tessellations in the Mogao Caves

Class
5th Grade Geometry

Standards

Organizing Questions
1. What is the process behind the Mogao caves?
2. What are the mathematical properties in the Mogao caves?
3. What kind of problems did the artists solve using mathematical thinking?

Introduction
The main goal of this activity is to incorporate California Common Core math standards into the process of creating the Mogao caves. The class will begin with a discussion of the symmetry and rotation found in tessellation artwork by MC Escher. We will relate the orientation of the Mogao caves to the x-axis and y-axis. The entrance of the Mogao caves faces East, the positive x-axis, and the back wall of the Mogao caves is West of the entrance. North is the positive y-axis, and South is the negative y-axis. Students will plot given ordered pairs as points on the coordinate plane. These points will be connected to form outlines and polygons, which will be painted to illustrate a tessellation. Students will then create their own tessellation on the coordinate plane, providing a list of the ordered pairs used.

Objectives
1. Learn about the Mogao caves and the process behind creating the murals.
2. Analyze tessellations.
3. Apply knowledge about the Mogao caves and cardinal directions to the coordinate plane.
4. Create art using knowledge of coordinate planes and tessellations.
5. Discuss the coordinate plane using cardinal directions.

Materials:
1. Empty coordinate plane, 2 copies per student for homework assignment + 2 copies per group for class assignment
2. Homework assignment instructions, 1 copy per student
3. Levels 1, 2, and 3 Artist project instruction templates, 2 sets per student + 1 set per group
4. Black markers, 1 per student
5. Poster paint (blue, green, teal, red, yellow, brown, and white), sufficient quantities for full class use
6. Paintbrushes, at least 1 per student
7. PowerPoint presentation with photos and directions

Equipment
1. Projector
2. Blank whiteboard
3. Printer
4. Internet access

Teacher preparation

1. Create PowerPoint.
2. Set up projector and test to make sure everything functions correctly.
3. Create graphic organizer for warm-up, vocabulary words, and info.
4. Create and print instructions for Levels 1-3 Artist class project.
5. Create and print instructions templates.
6. Create and print coordinate plane worksheet.
7. Buy paint, brushes, and black markers.
8. Have students read "The Artist's Tale" from *Life Along the Silk Road* by Susan Whitfield, emphasizing passages describing the process of creating the Mogao caves' murals.
9. Ensure students have a basic understanding of number lines and integers. Review these concepts briefly before beginning the lesson.

Time

One 50-minute class period

Procedures

1. Warm-up: Think / Pair / Share discussion of a painting by MC Escher
    a. Pass out vocabulary sheet for discussing tessellations. Make sure your vocabulary sheet includes words that will enable your students to understand and discuss tessellations, i.e. symmetrical, rotating, etc.
    b. Project an image of a painting by MC Escher on the white board for your students to refer to.
    c. Pair students up to discuss the painting with each other using the words on their vocabulary sheet.
    d. After students have had the opportunity to discuss in pairs, begin a class discussion of the painting and explain the concept of tessellations.
2. Introduction to Lesson
    a. Project a diagram of the Mogao caves clearly showing the entrance facing East and the phases of the sun as it rises and sets. Use this diagram to introduce the horizontal x-axis, where East represents the positive x-axis and West represents the negative x-axis. Let North represent the positive y-axis and South represent the negative y-axis
    b. Give a brief explanation of the way that artists used taut string to create straight horizontal and vertical lines on the cave wall. Connect the artists' linear grids to the coordinate plane and their use of pounces to ordered pairs.
    c. Show the class photos of the geometric designs on the Mogao cave roofs as examples of tessellations.
3. Activity: Level 2 Artists project
    a. Invite students to imagine that they are artists at the Mogao caves.
    b. Divide students into pairs.
    c. Pass out the instruction sheet for the Level 2 Artists project and the blank coordinate plane graphs, ensuring that each pair has one copy of each.
    d. Instruct students to plot the coordinate points on their instruction sheets on their blank coordinate plane graphs. Students must connect the coordinate points in the order in which they were plotted. Demonstrate the process by plotting 2 points and then connecting them together as a class.
    e. After students have finished plotting and connecting the coordinate points, have pairs compare their results to ensure that everyone has followed the instructions completely and correctly.

    f. Pass out paintbrushes, paints, and the second set of instructions on coloring the shapes that they have just drawn.
    g. Ask students to follow the instructions to fill in the shapes they have created with the appropriate color.
    h. After the students have finished coloring their shapes and the paint has dried, have students place their coordinate planes together on one large sheet to create a tessellation. This is similar to a gallery walk, in which students can check and compare their work. You may need to ask students to explain their process to their partner in order to check for mistakes and understanding.
4. Exit slip: Master Artists project
    a. Have students inhabit the role of Master Artist by creating their own artwork on the coordinate plane.
    b. Students' designs must show symmetry and rotation.
    c. Each pair of students must create two sets of instructions for replicating their coordinate plane. These instructions should include a list of ordered pairs, an explanation of which points to connect with lines to form polygons, and colors for each polygon.
    d. At the end of class, each student pair should exchange their set of instructions with another pair of students' set of instructions. Each student should leave class with a set of instructions by a classmate.
5. Homework
    a. Pass out a blank coordinate plane graph to each student.
    b. Have students use the instructions that they just received from their classmates to recreate the geometric artwork that their classmates designed.

Assessment

1. Level 2 Artists class project
2. Homework assignment

COORDINATES IN CAVES

Coordinate planes in the context of the artistic process behind the Mogao cave murals.

Vivian Lam (Lum Ho Yun)

Summer Institute of China, 2015

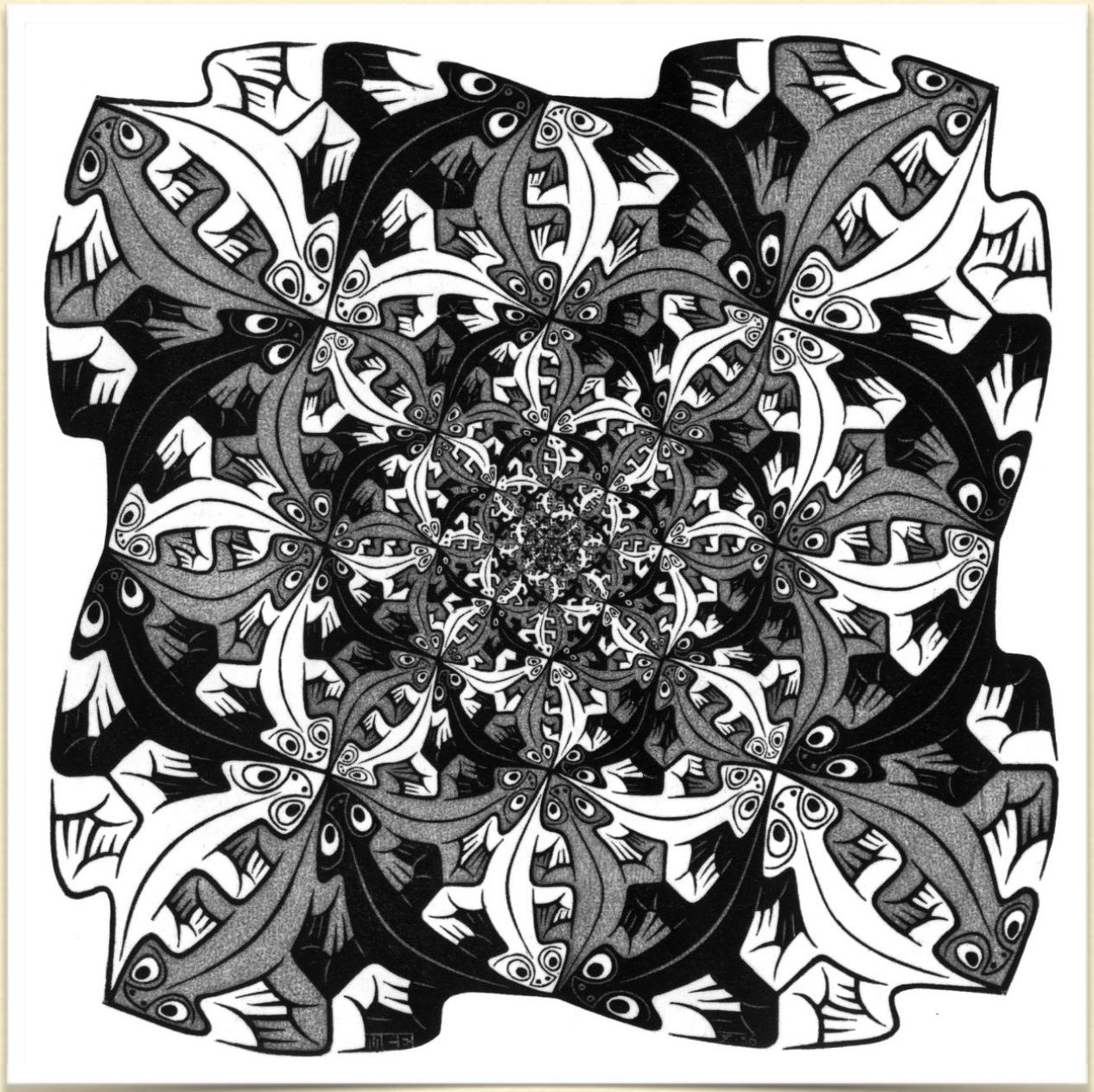
California Common Core State Standards
Grade 5 Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

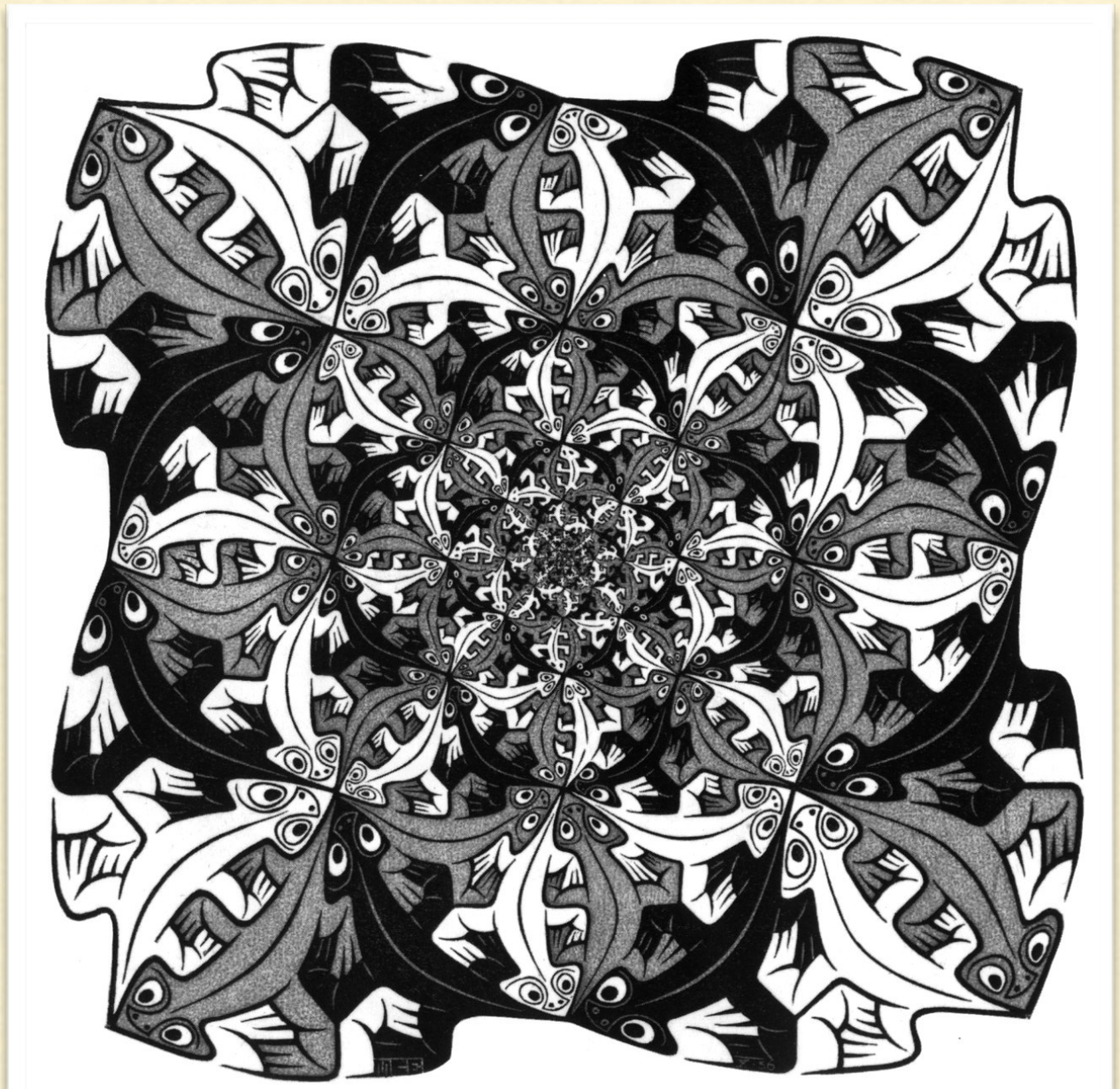
2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

WARM UP: THINK



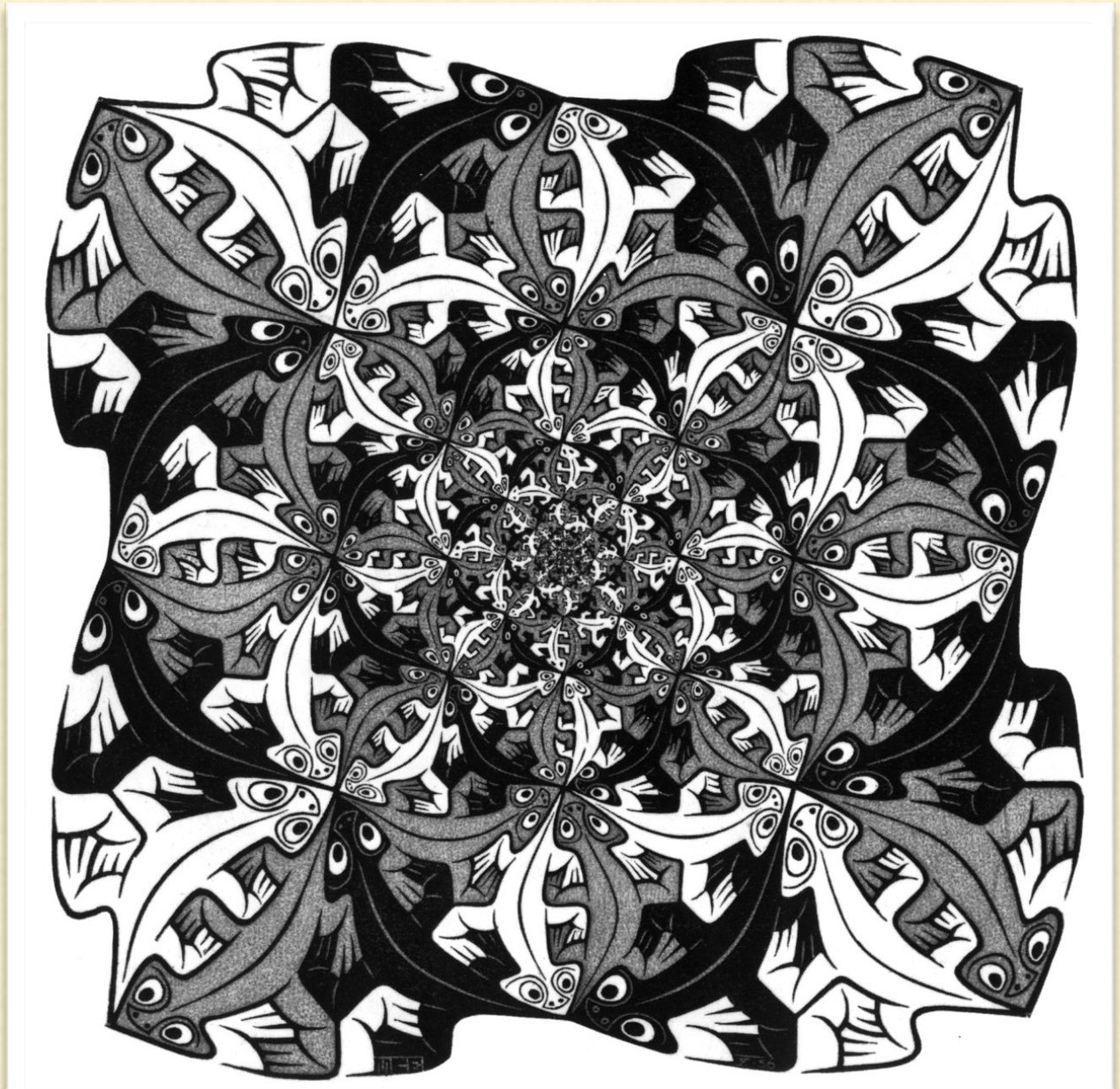
- What is special about this artwork?

WARM UP: PAIR



- Pair up with a person sitting next to you.
- What is special about this artwork? Discuss it using these words:
  - **axis: (n)** an imaginary line
  - **symmetrical: (adj)** having the same parts across an axis
  - **rotating (v)** turning around on an axis
  - **tessellation: (n)** pattern made of repeating identical shapes, fitting together with no overlap or gaps.

WARM UP: SHARE



- What is special about this artwork? Share your partner's thoughts with the class using these words:

  - **axis: (n)** an imaginary line
  - **symmetrical: (adj)** having the same parts across an axis
  - **rotating (v)** turning around on an axis
  - **tessellation: (n)** pattern made of repeating identical shapes, fitting together with no overlap or gaps.

MOGAO CAVE ART

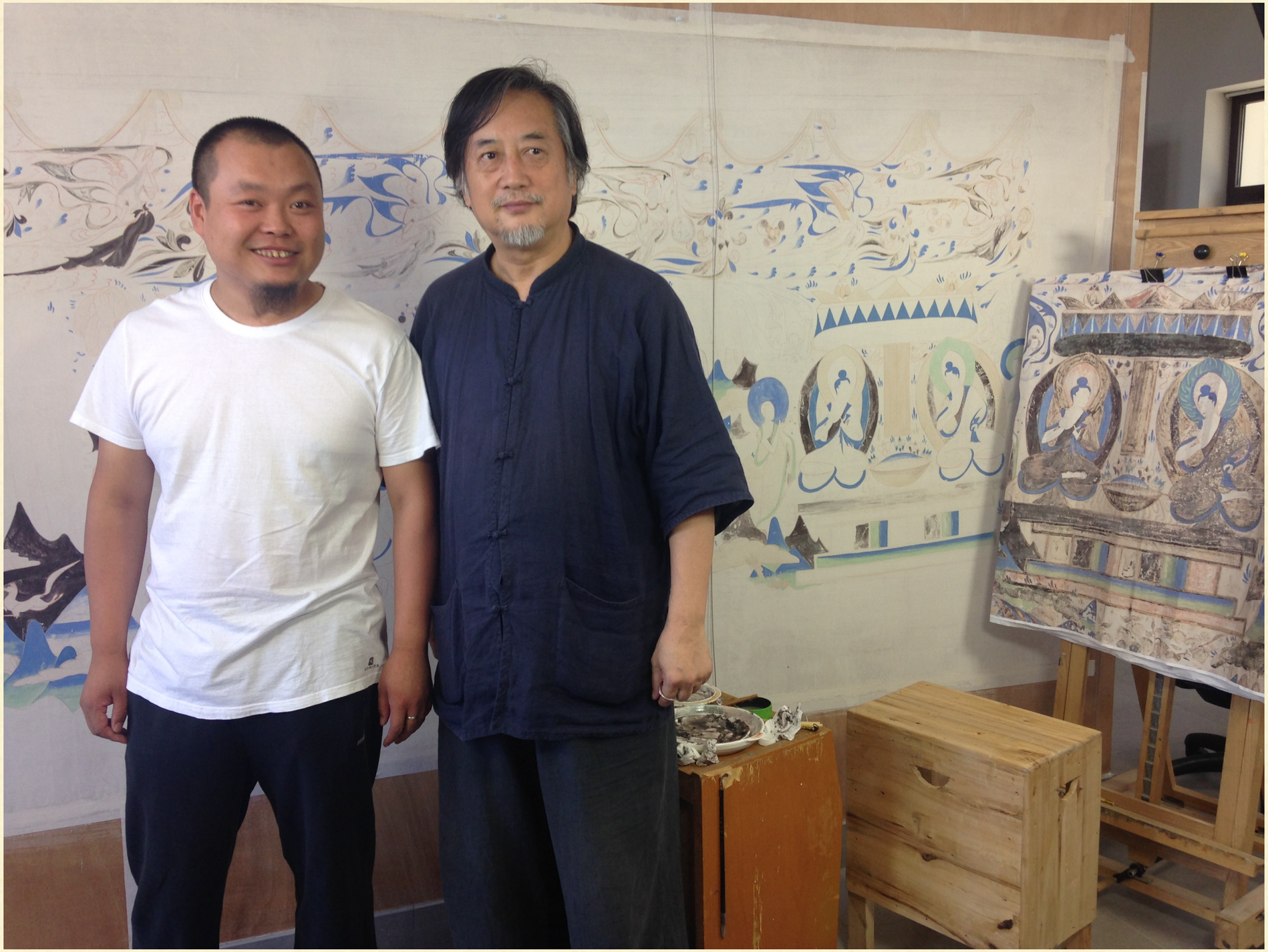
Noble families sponsored the caves. Community members, merchants, and artisans built the caves together.

MOGAO CAVES

- Dunhuang, China
- Apricots and grapes
- Edge of the Gobi Desert
- Buddhism
- 4th to 14th century
- Damage and deterioration

CAVE 45 REPLICA

Conservation and replication efforts have been led by Getty Institute of Conservation and Dunhuang Academy.

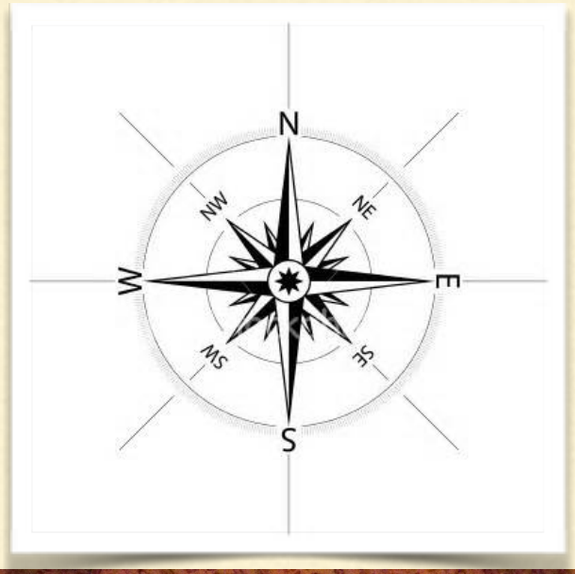


Yue Yang, an artist working on a replica of a Mogao Cave. He worked for 4 years to be accepted by Dunhuang Academy. The art behind him has been in progress for 2 years, and may take a total of 3 years to complete.

FACING EAST

The mouth of every Mogao cave is open to the east, where the sun rises in the morning.
The back of the Mogao cave is towards the west, where the sun sets in the evening.

FACING EAST



The mouth of every Mogao cave is open to the east, where the sun rises in the morning.
The back of the Mogao cave is towards the west, where the sun sets in the evening.

MOGAO CAVE 406 CIELING

- What is special about this art?
  - **axis: (n)** an imaginary line
  - **symmetrical: (adj)** having the same parts across an axis
  - **rotating (v)** turning around on an axis
  - **tessellation: (n)** pattern made of repeating identical shapes, fitting together with no overlap or gaps.
- How do you think the artists created this?
- What do you think the artists used?

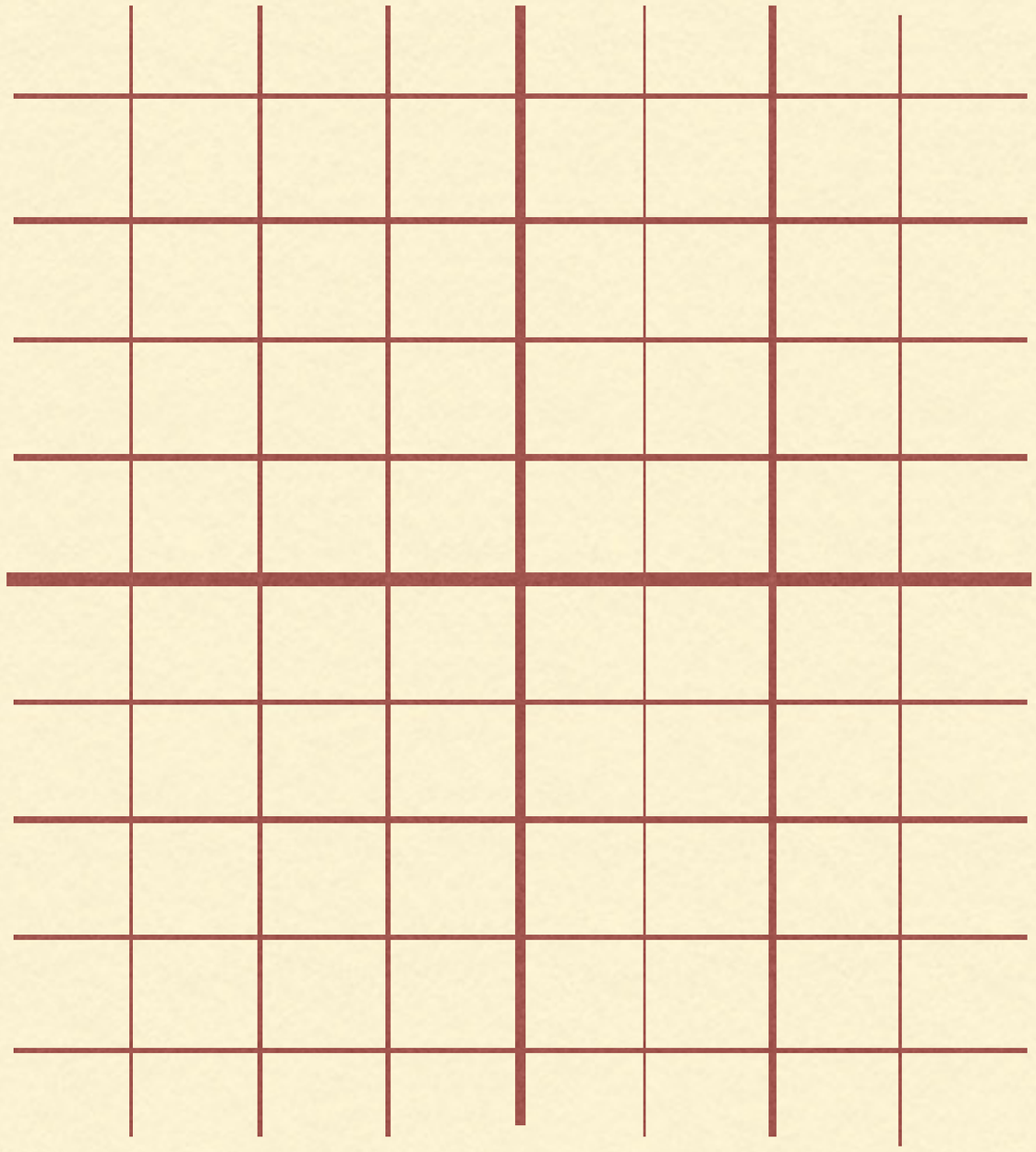
ARTISTIC PROCESS

Artist: Fan Li Juan, Dunhuang Academy of Art

- Freehand sketch: Artists divided the walls into large areas and drew a sketch using earthen red pigment.
- Drawing from a draft: Artists create a giant grid on the wall, then redraw the draft picture onto the wall.
- Pounces: Artists used special stencils called pounces to paint the caves.

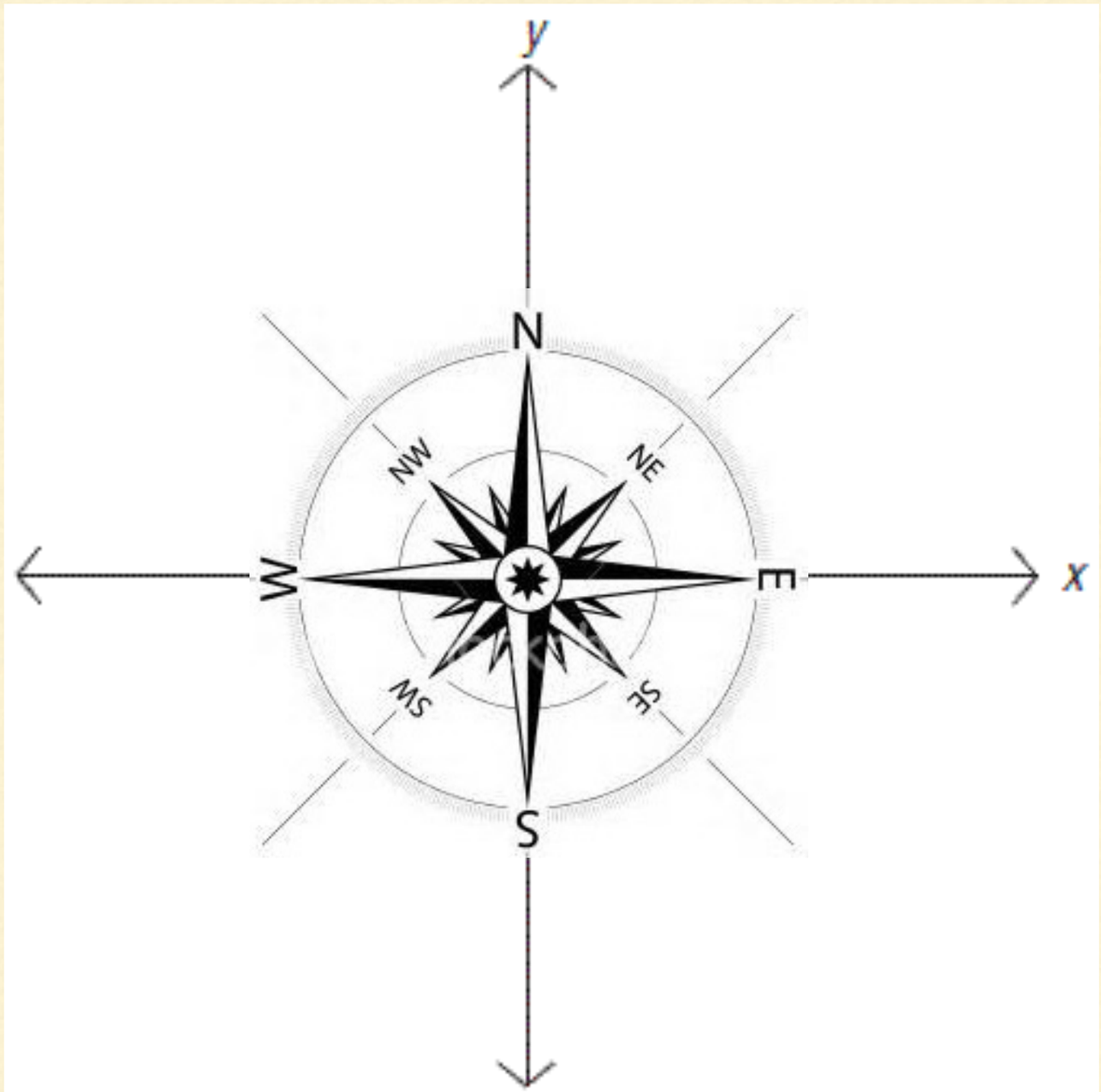
LEVEL 1

his young apprentices to mark up the wall. To do so they dipped string in red powder, held it taut near the wall and then flicked it so that the powder transferred itself from the string to the wall, thus defining the limits of each horizontal register and of each main composition. The main events of the story



- Artists create a giant grid using lines of red powder, beginning with a horizontal register.
- "The Artist's Tale," which all of you should have read by now...
- How did the artists use this grid?

1. COORDINATE PLANE



X-axis:

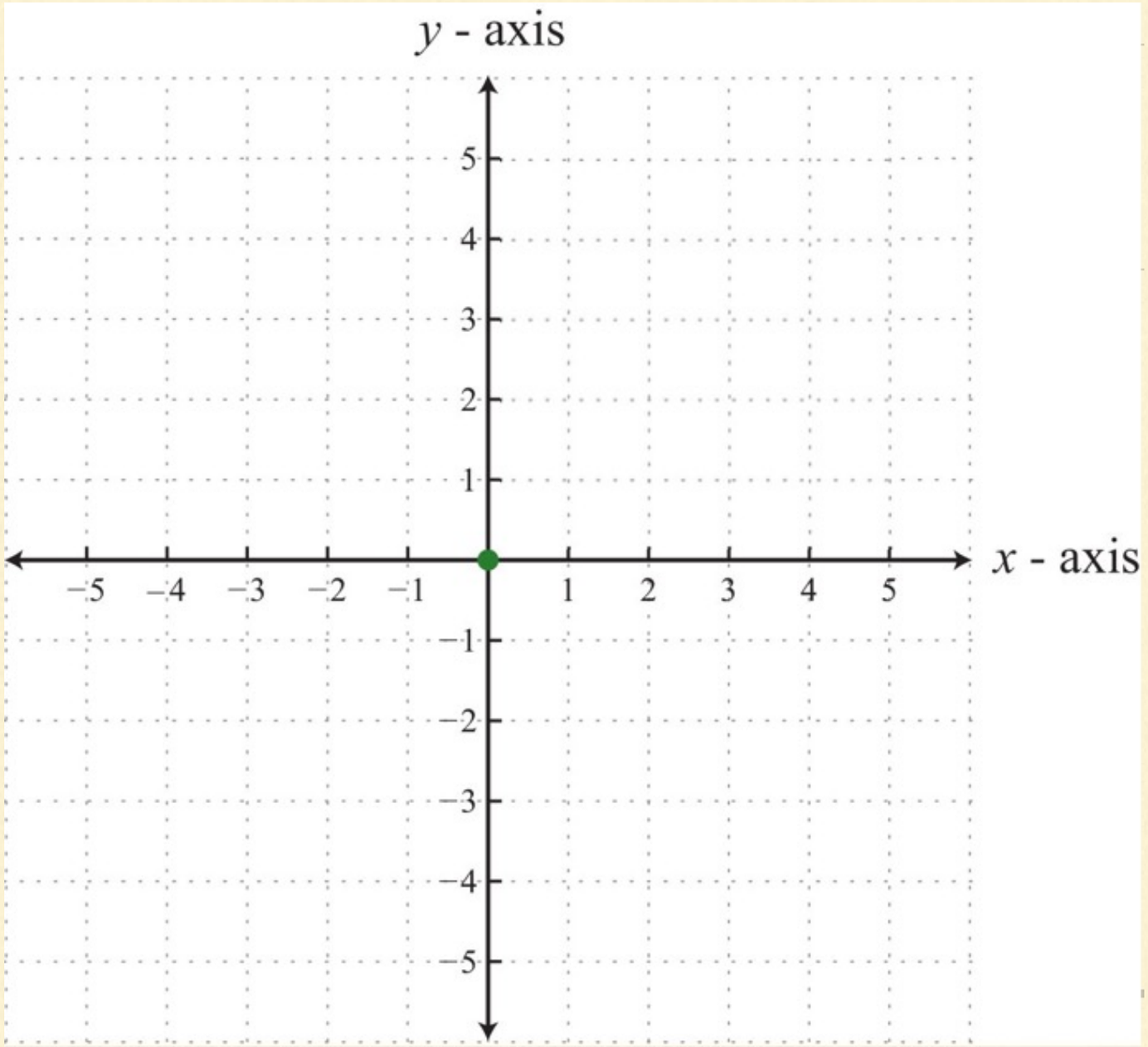
- East
- West

Y-axis:

- North
- South

Using your knowledge of number lines, which direction is positive on the x-axis? ...one the y-axis?

1. COORDINATE PLANE



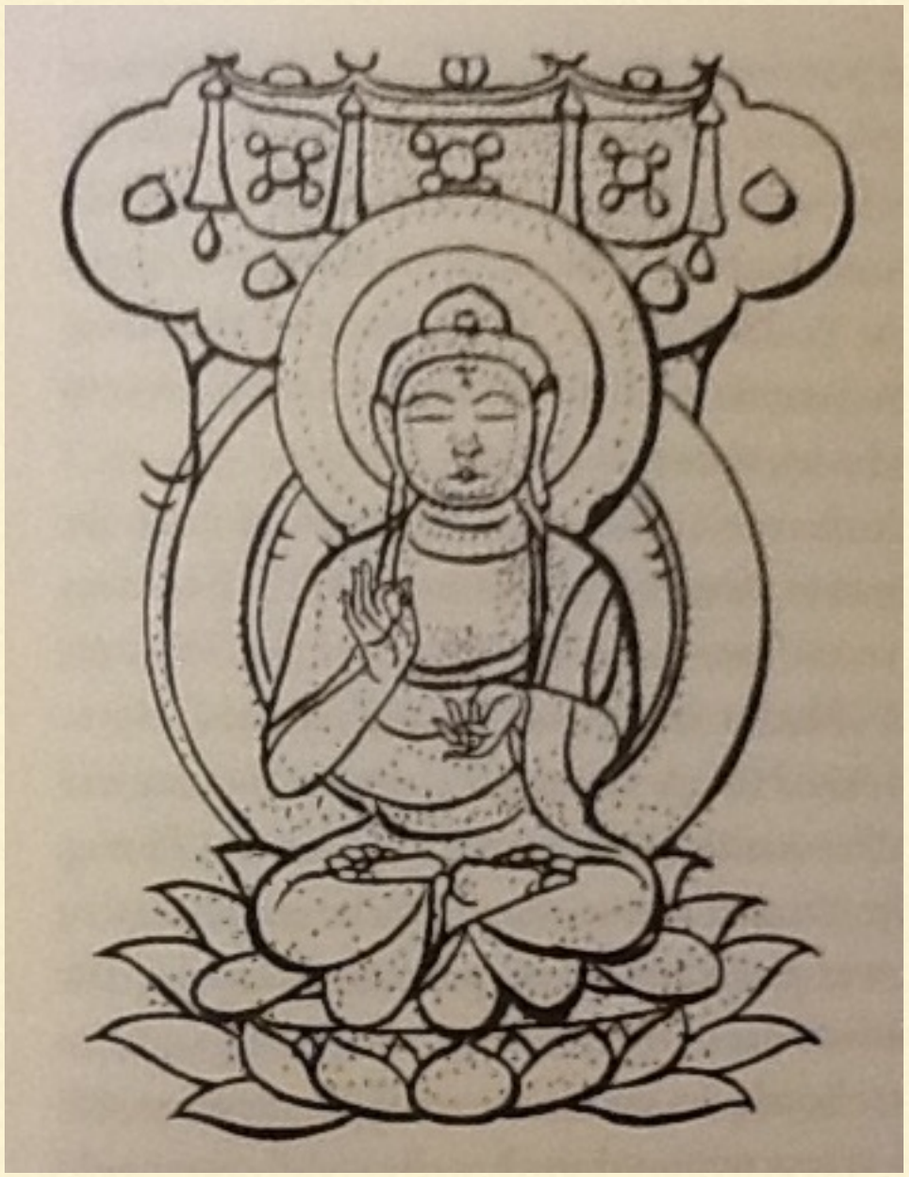
X-axis:

- East: positive
- West: negative

Y-axis:

- North: positive
- South: negative

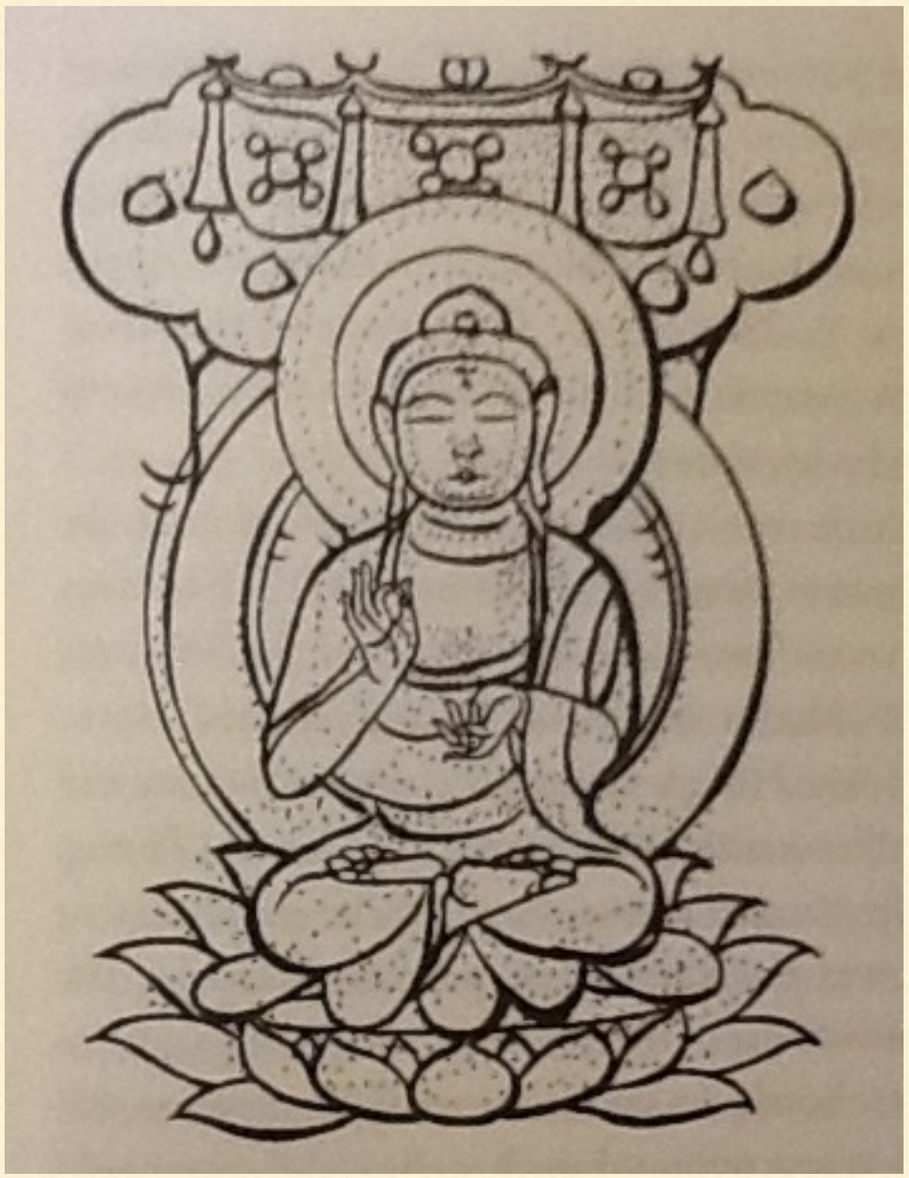
LEVEL 2



- How was this outline created?

corner segments between each sloping wall. Then Baode had to consider the lower registers and the coffered ceiling. It was common to use stencils or pounces for the latter in order to ensure that the design was repeated uniformly. The outline of the design was drawn in an ink wash on squares of thick, coarse local paper and then pricked through with a needle. Next, the artist would fill a cloth bag with red cinnabar powder, tie it up, and attach it to the end of a long stick; scaffolding had been erected inside the cave so that the ceiling was accessible. With one hand holding the paper pounce in place on the ceiling, the artist would bang the powder bag against the paper so that, on impact, powder leaked through the rough cloth and the holes in the paper onto the ceiling behind, leaving a faint red outline. This process was continued until the whole ceiling was covered in red lines. For this particular

LEVEL 2

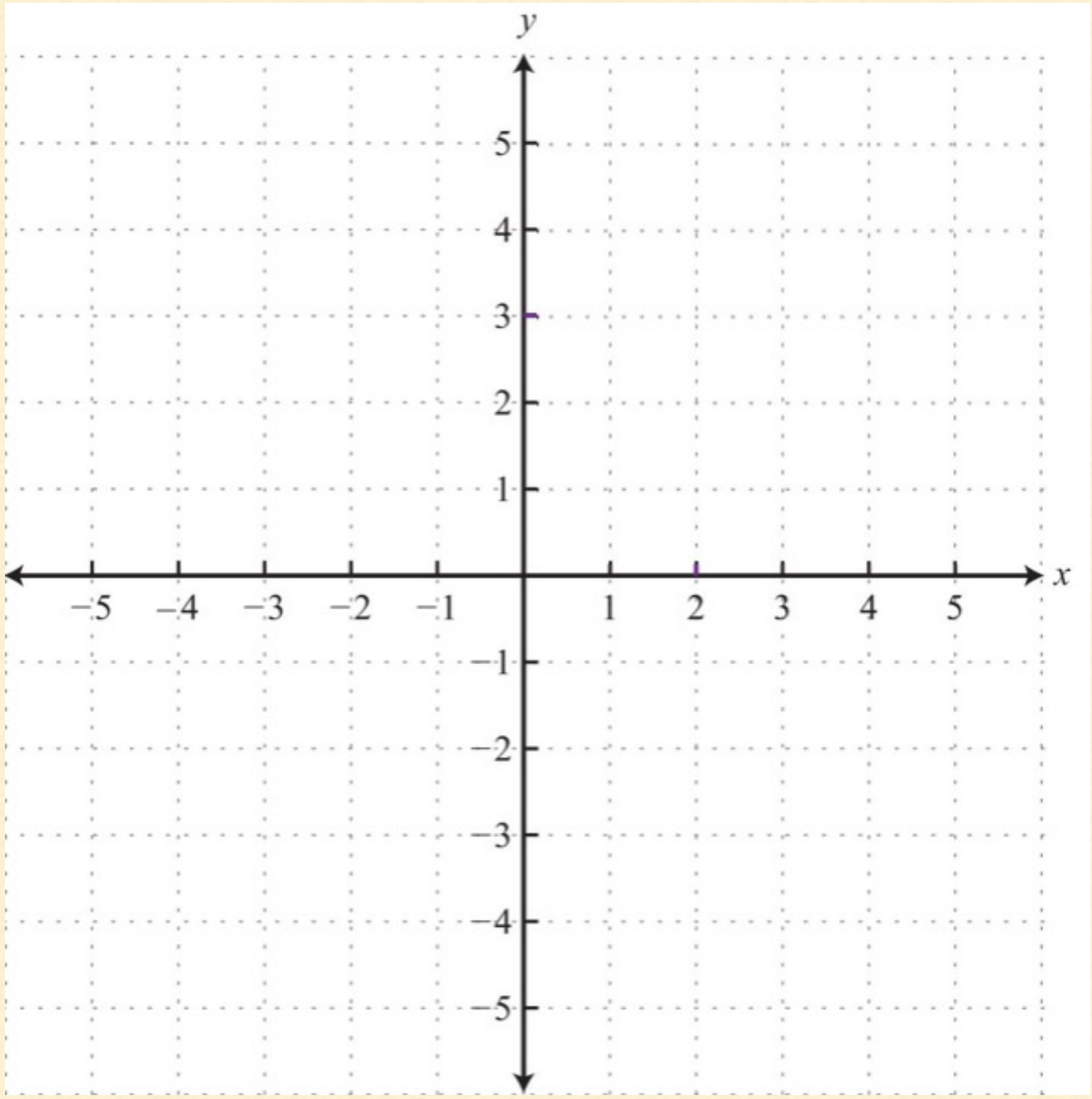


- How was this outline created?

1. Points plotted.

2. Points connected by lines, which forms shapes.

2. ORDERED PAIR



- Ordered pair (**X** , **Y**) provides the directions to a point on the coordinate plane (think of a GPS).

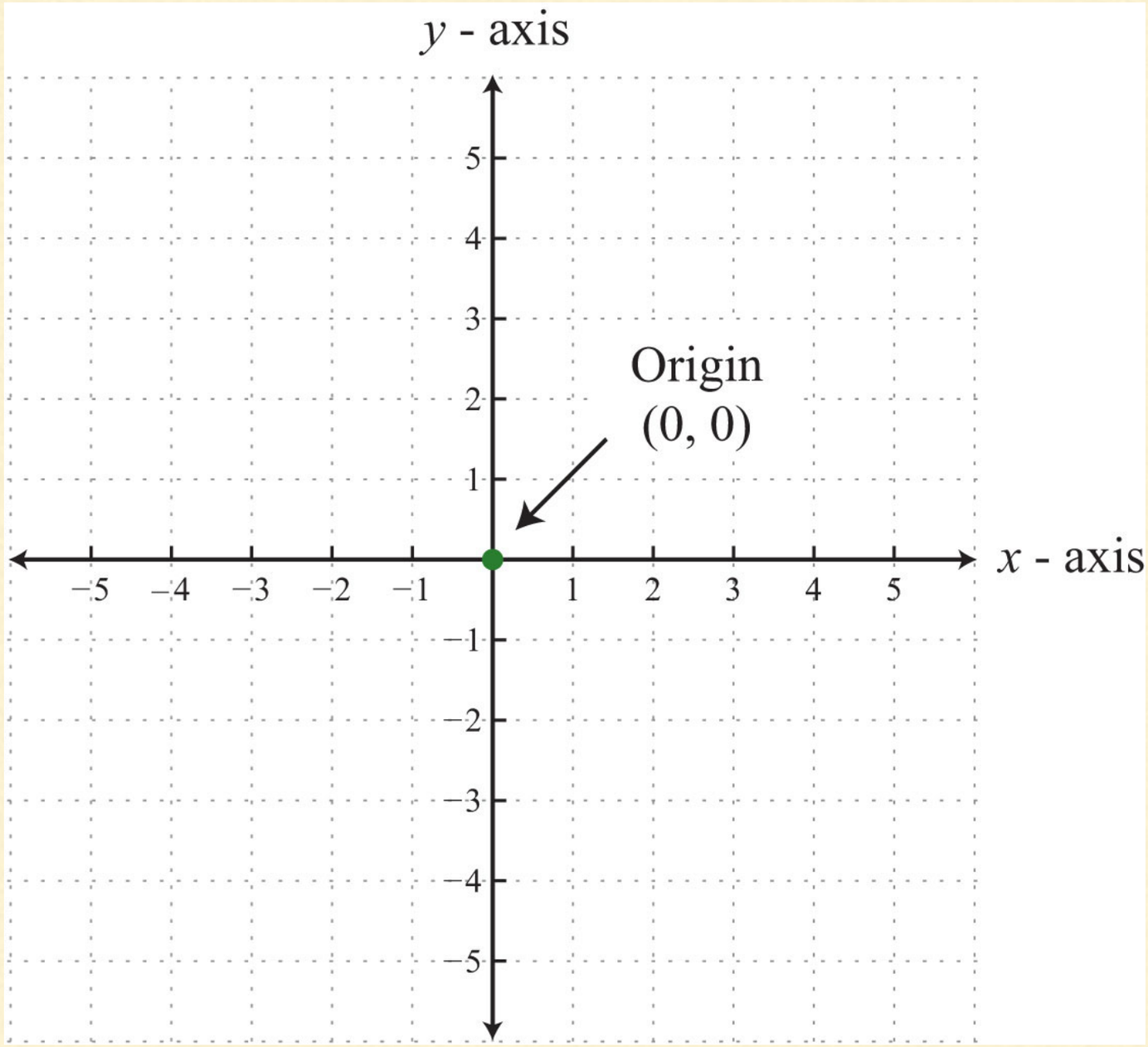
- **X-coordinate**:

  Number of units to move on the x-axis,
  East (positive horizontal direction) or
  West (negative horizontal direction)

- **Y-coordinate:**

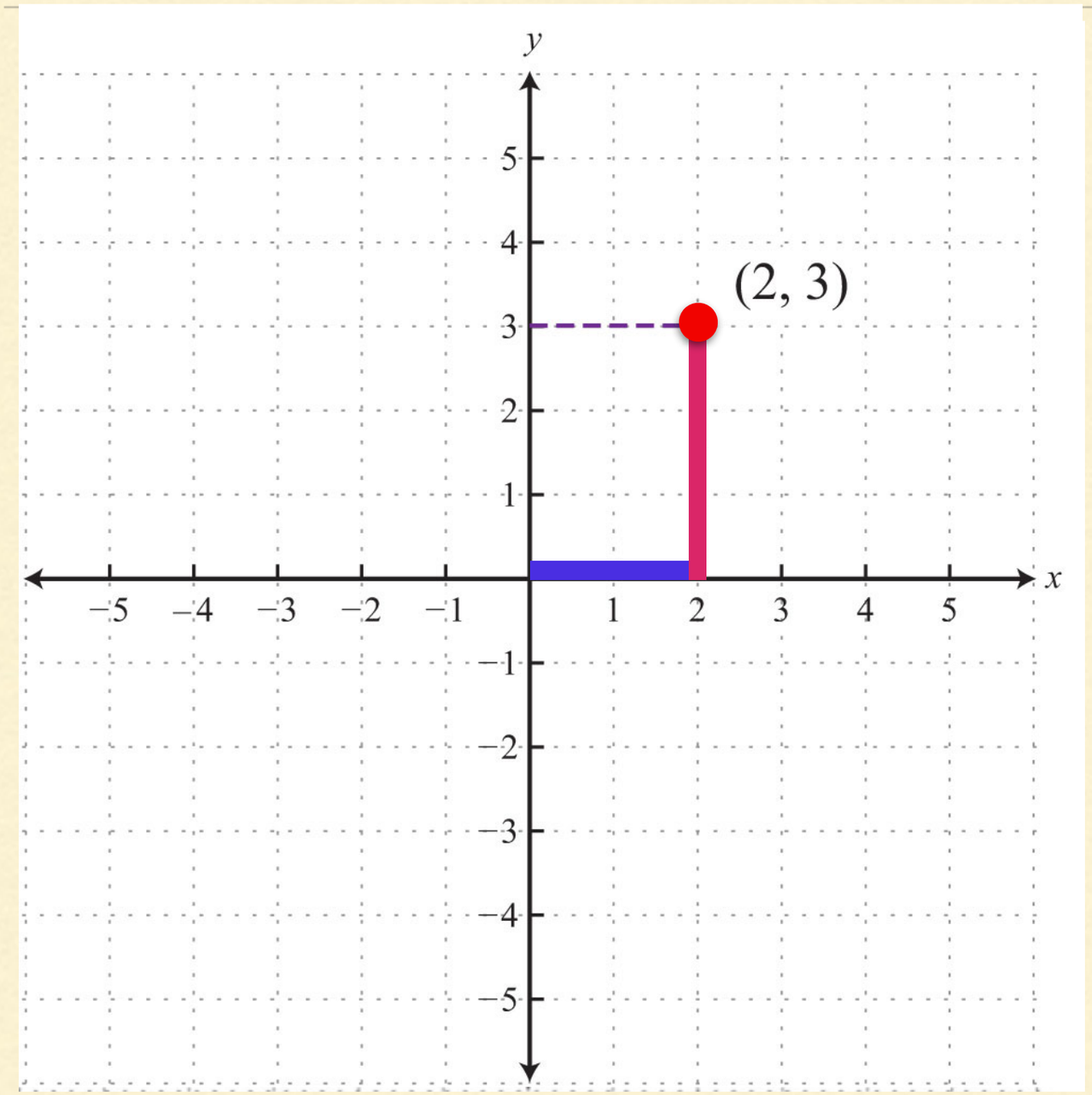
  Number of units to move on the y-axis,
  North (positive vertical direction) or
  South (negative vertical direction)

3. ORIGIN



- (**X** , **Y**) = (0 , 0)
- **X-coordinate**: move 0 units
- **Y-coordinate**: move 0 units
- The ordered pair (0, 0) is a point called the **Origin**.
- It is where the x-axis and y-axis intersect, and **where all points begin.**

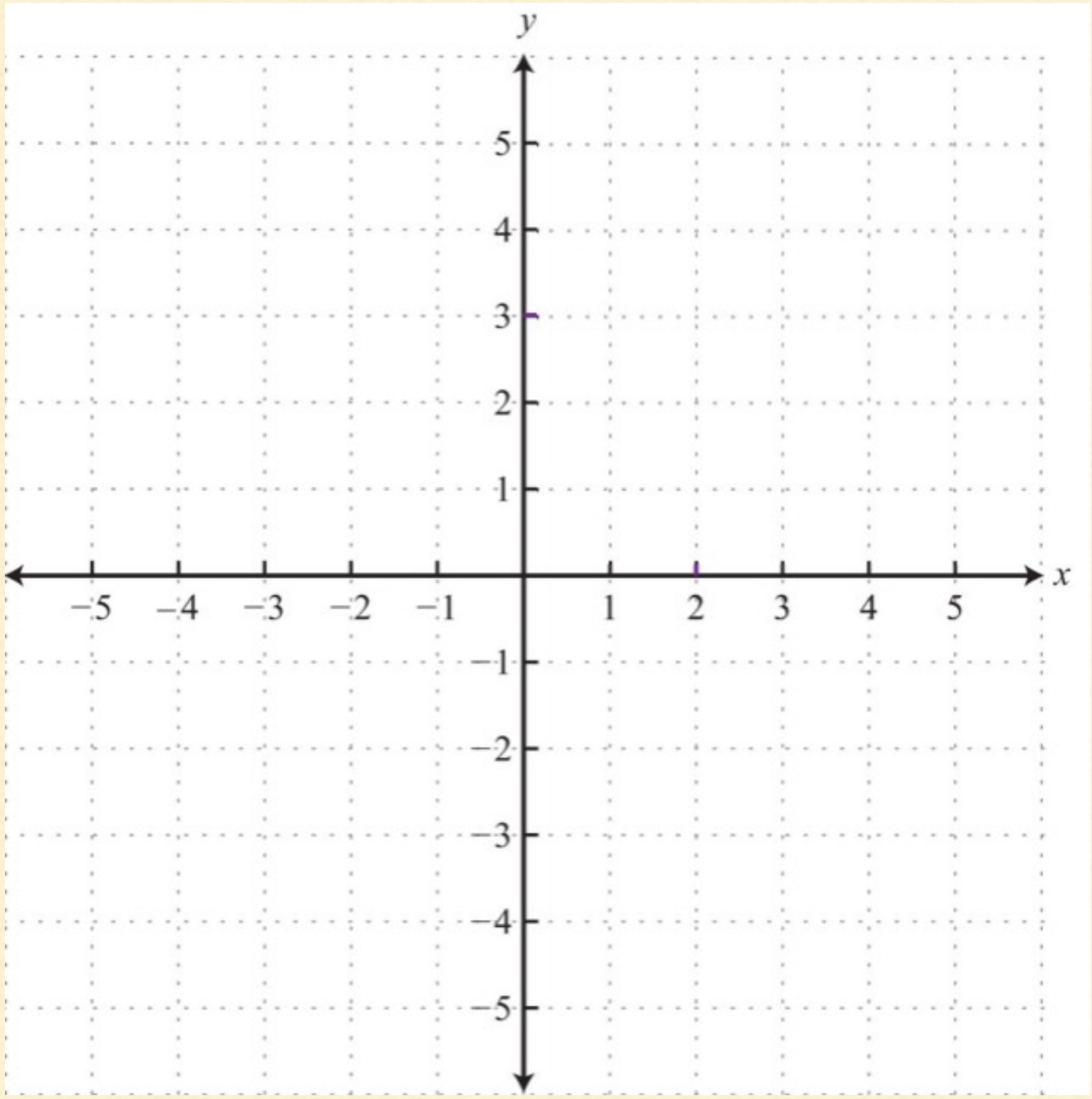
EX:



- (**X** , **Y**) = (2 , 3)

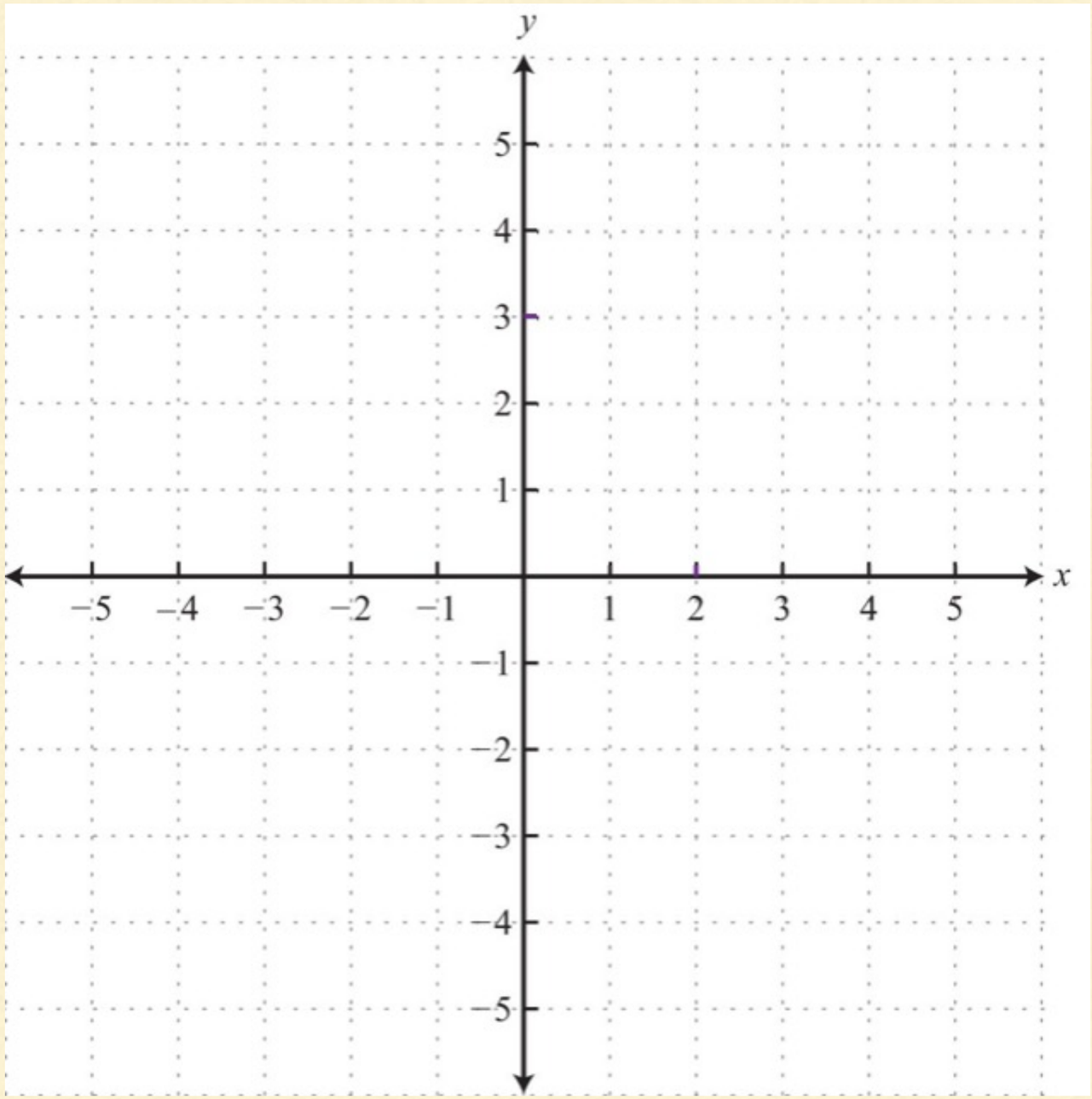
- **X-coordinate**: __2__ units __right__
- **Y-coordinate**: __3__ units __left__

APPRENTICES IN ORDERED PAIRS



- Noble donors Karen and Cherie have commissioned a cave.
- You and your partner will work together as apprentices to replicate the Master Painter's design with their instructions.

LEVEL 2: PLOTTING POINTS



Connect each group of points in the order given to form a shape.

(**X** , **Y**) = East/West , North/South)

- Shape A:

  (1, 1) (1, -1) (-1, -1) (-1, 1)

- **Shape B:**

  (1, 1) (3, 0) (1, -1) (0, -3) (-1, -1) (-3, 0) (-1, 1) (0, 3)

- Shape C:

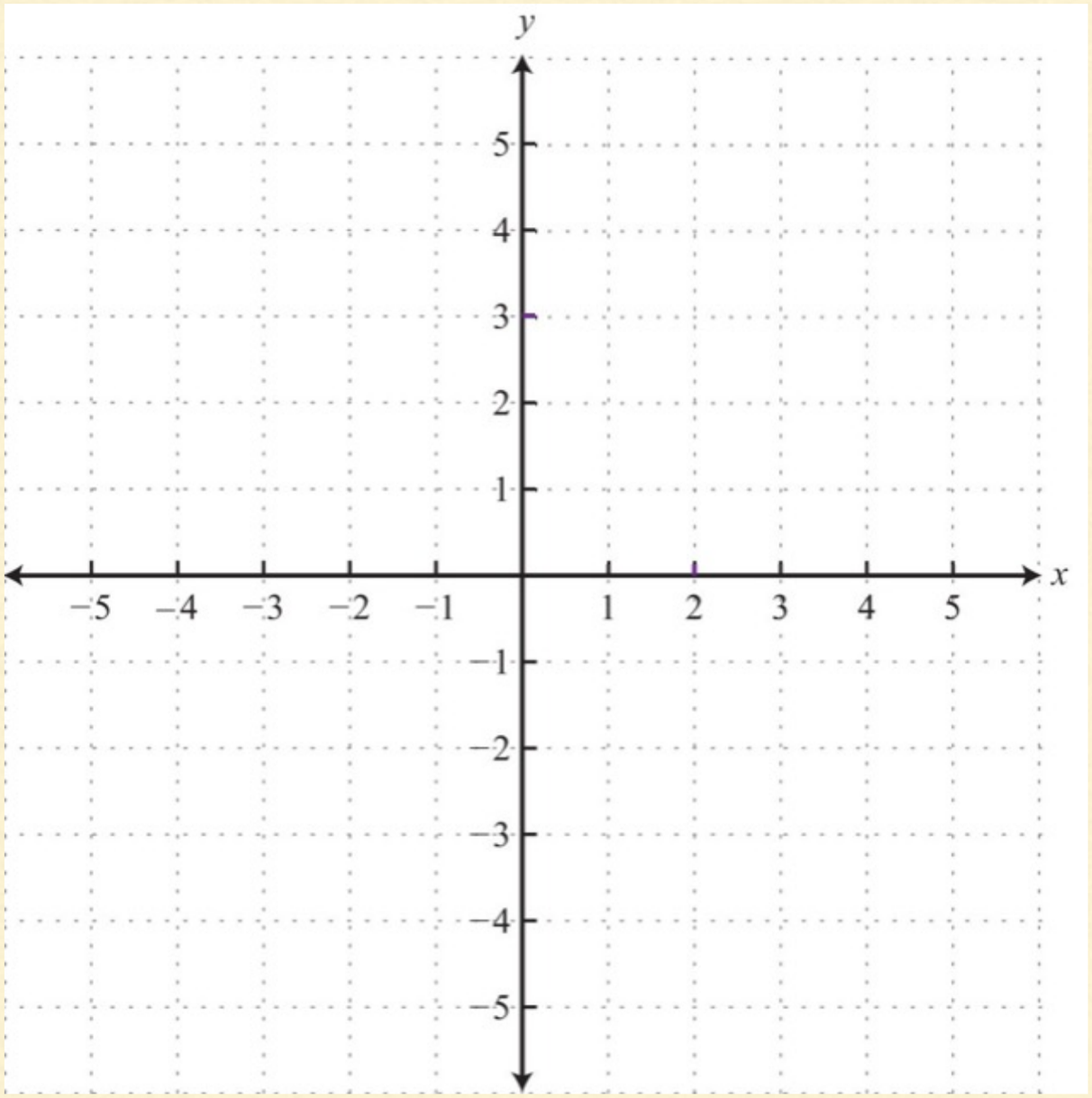
  (4, 4) (4, -4) (-4, -4) (-4, 4)

- Shape D:

  (5, 5) (4, 2) (5, 0) (4, -2) (5, -5) (2, -4) (0, 5) (-2, -4)

  (-5, -5) (-4, -2) (-5, 0) (-4, 2) (-5, 5) (-2, 4) (0, 5) (2, 4)

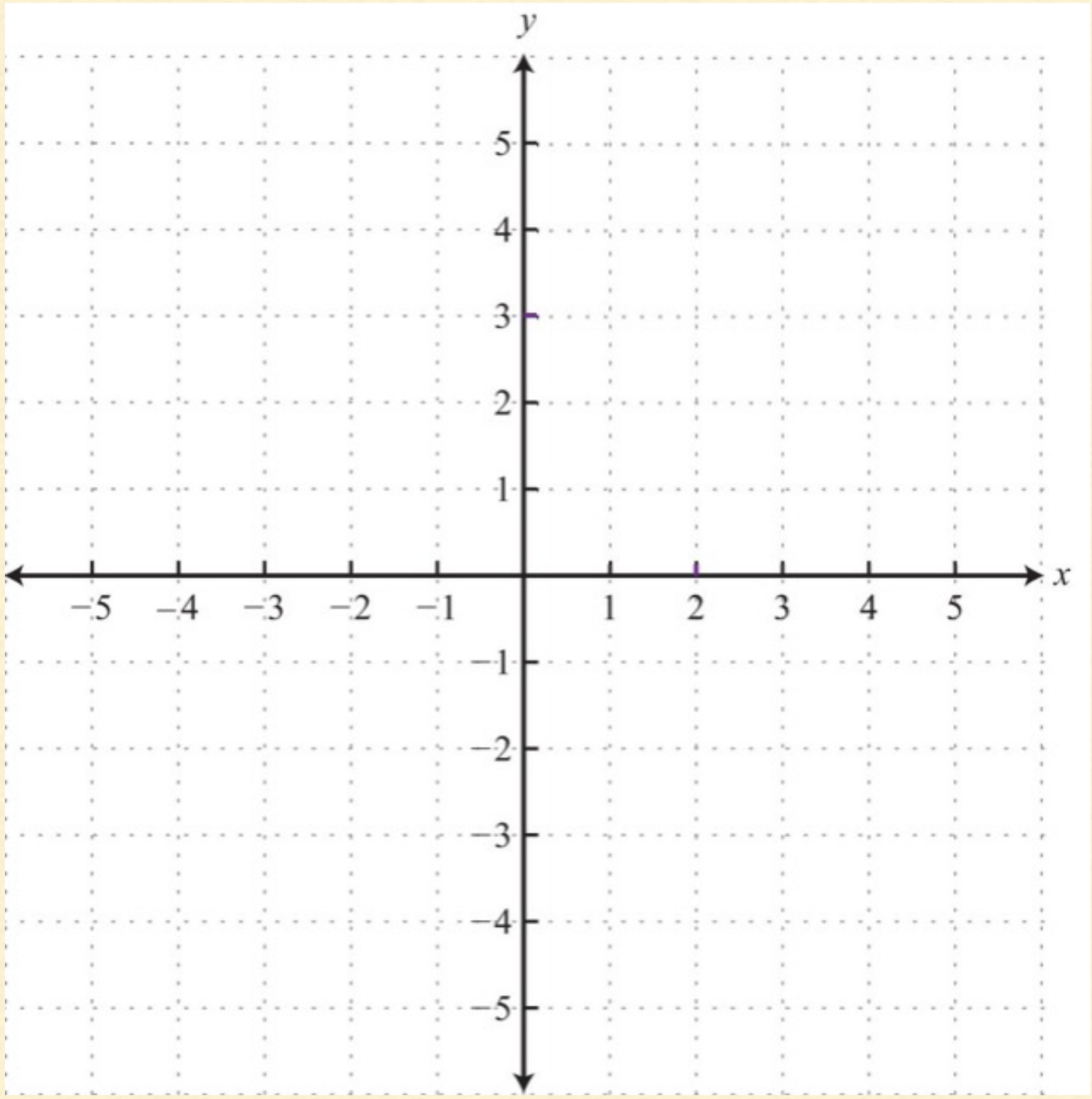
LEVEL 2:



- Compare your points with another group.
- Did you get the same points?
  - Yes: Outline the shapes in black paint.
  - No: Find out why the points are not the same, then find the correct points.

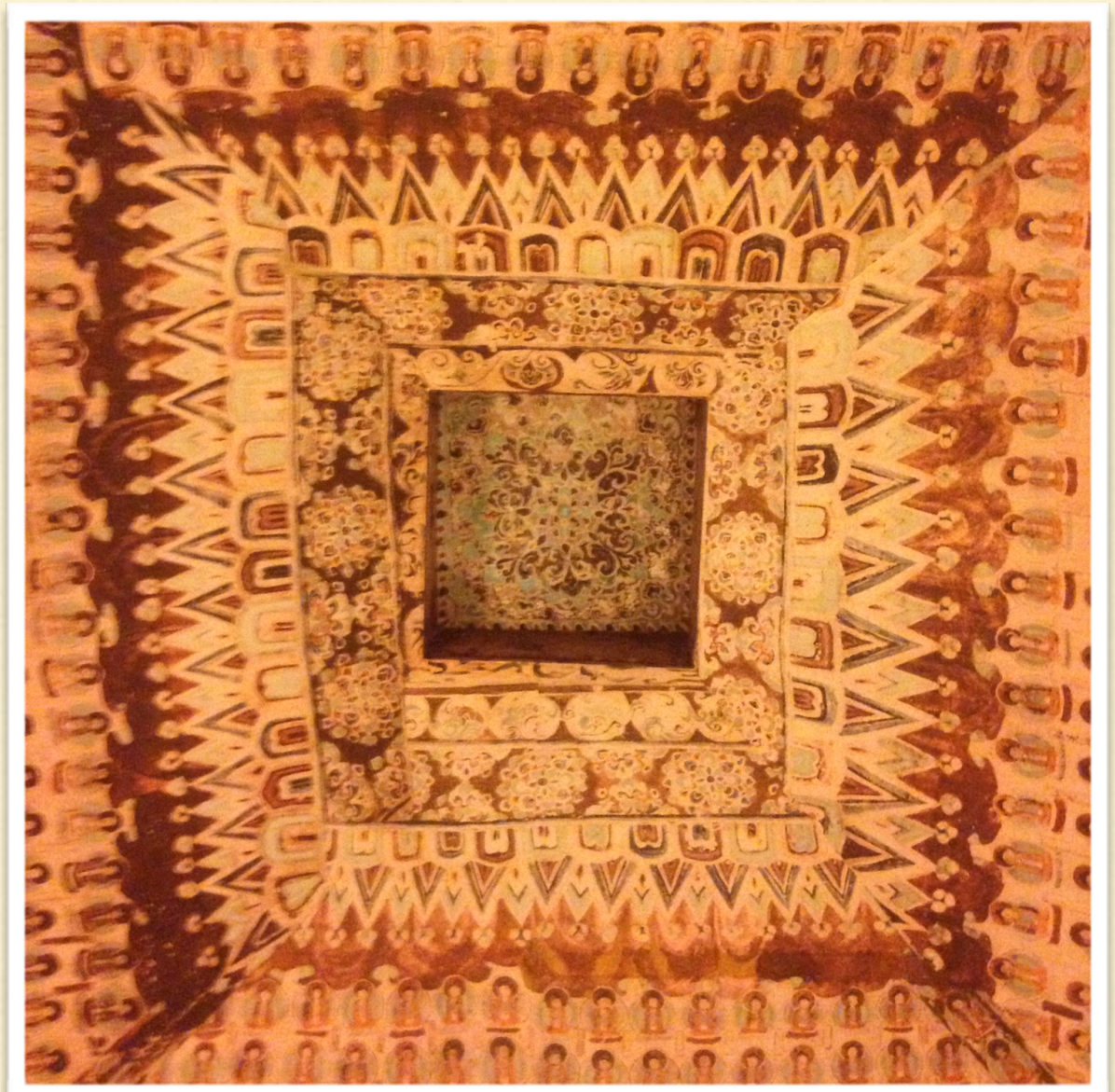
LEVEL 3:

OUTLINING & PAINTING WITH ORDERED PAIRS



- Shape A: Blue Paint
- Shape B: Red Paint
- Shape C: Yellow Paint
- Shape D: Red Paint
- Edge: Blue Paint

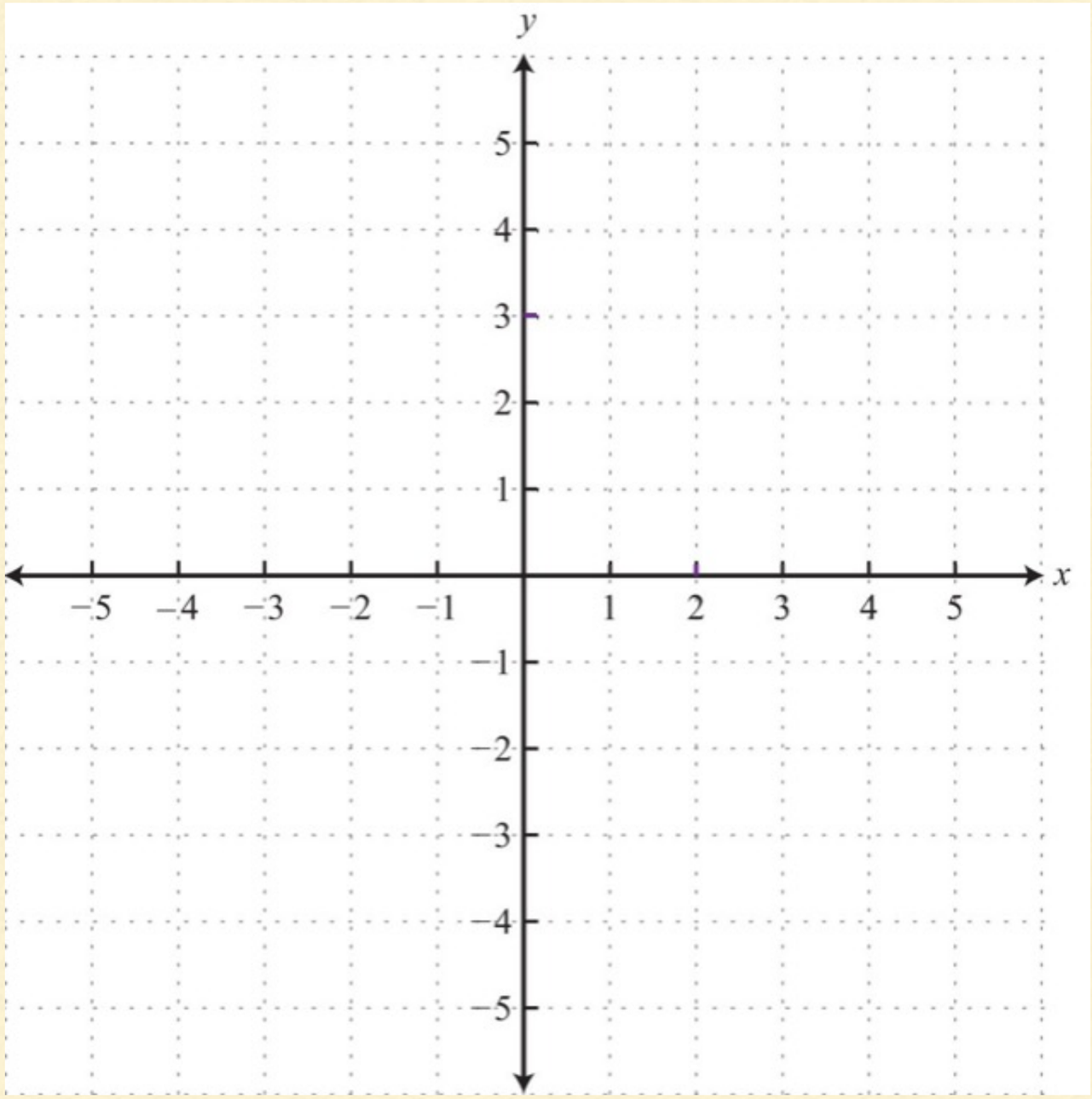
QIAOJIANG (MASTER PAINTER)



- You and your partner are now Master Painters, or Qiao Jiang.
- Work together to create a design that has symmetrical and rotating patterns.
- Create instructions for Level 2 apprentices to replicate it.
    1. List of ordered pairs.
    2. Directions for connecting lines.
    3. Colors for each shape.

Material: Instruction Template, Coordinate Plane

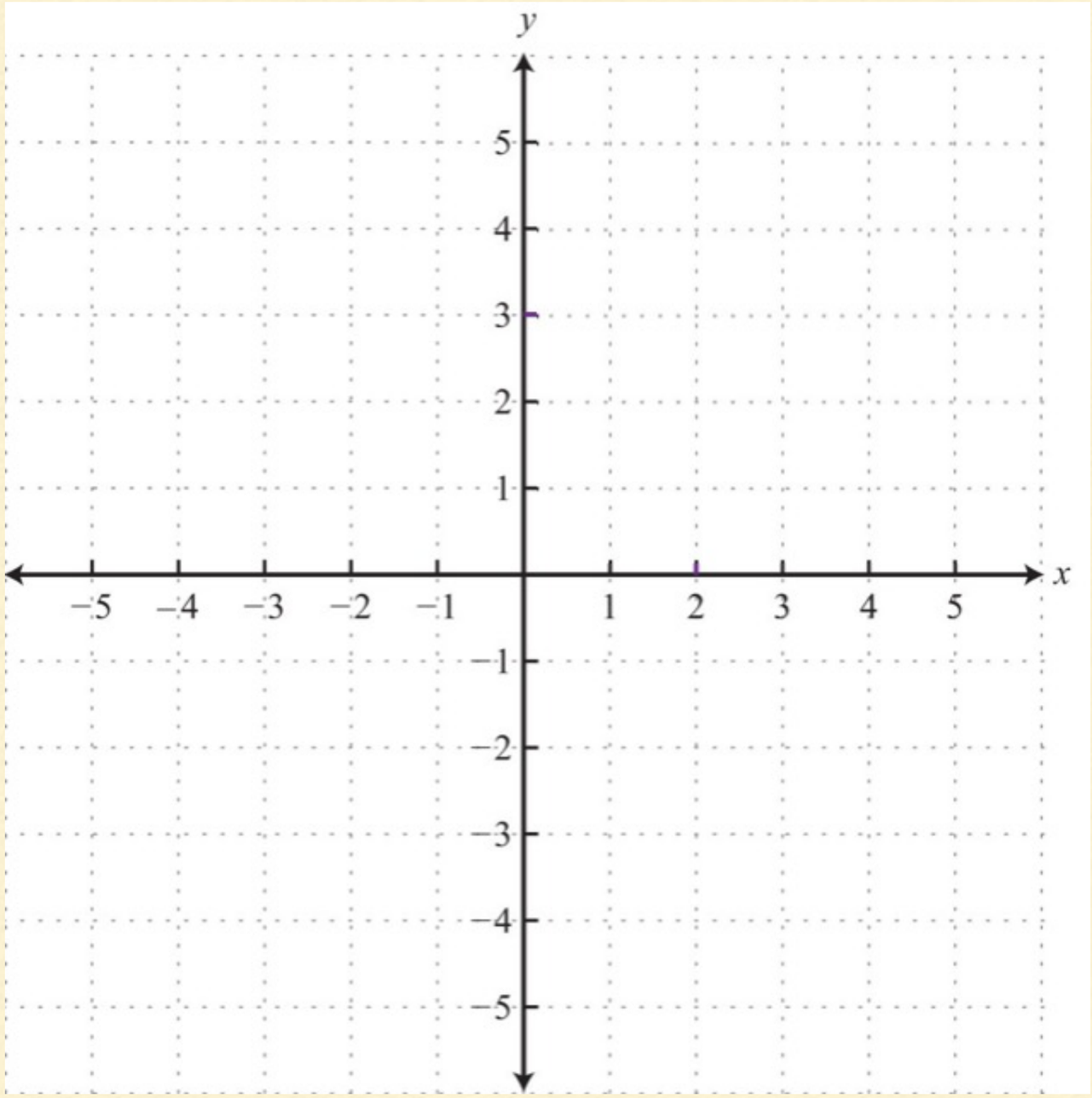
FINISHING UP



- When you and your partner have finished painting, please:

1. have one person bring the art and instructions to Ms. Vivian.
2. have one person in your group clean the brushes and place them in its container.
3. have one person in your group will clean the table.
4. have one person throw any trash away in bin.

HOMEWORK



- You have been given instructions to replicate the artwork for a cave by other Master Painters. Please complete it at home.

- Create a piece of art on the coordinate plane. The art can be of anything, but must be school appropriate. Write instructions for other people to replicate your art.